

Chapter 4

SOME APPLICATIONS OF BINARY PULSARS TO FUNDAMENTAL PHYSICS

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Abstract

Binary systems containing at least one radiopulsar are excellent laboratories to test several aspects of fundamental physics like matter properties in conditions of extreme density and theories of gravitation like the Einstein's General Theory of Gravitation (GTR) along with modifications/extensions of it. In this Chapter we focus on the perspectives on measuring the moment of inertia of the double pulsar, its usefulness in testing some modified models of gravity, and the possibility of using the mean anomaly as a further post-Keplerian orbital parameter to probe GTR.

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1. Introduction

In this Chapter we discuss some applications of astrophysical binary systems containing at least one radiopulsar to fundamental physics. In particular, in Section 2. we investigate the perspectives on measuring the pulsar's moment of inertia (Iorio, 2009a; Kramer and Wex, 2009) through the extension of the general relativistic Lense-Thirring effect to the double pulsar system J0737-3039. Section 3. shows how the double pulsar can be used to put on the test some models of modified gravity (Iorio, 2009b). In Section 4. we deal with the possibility of using the mean anomaly as a further post-Keplerian parameter useful to test the gravitoelectric part of the General Theory of Relativity (GTR) (Iorio, 2007a); for a proposal concerning a new post-Keplerian parameter to test different aspects of gravitomagnetism with respect to the Lense-Thirring effect, see (Ruggiero and Tartaglia, 2005).

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2. The Moment of Inertia of Pulsar J0737-3039A: Prospects for its Measurement

The measurement of the moment of inertia I of a neutron star at a 10% level of accuracy or better would be of crucial importance for effectively constraining the Equation-Of-State (EOS) describing matter inside neutron stars (Morrison et al., 2004; Bejger et al., 2005; Lattimer and Schutz, 2005; Lavagetto et al., 2007).

After the discovery of the double pulsar PSR J0737-3039A/B system (Burgay et al., 2003), whose relevant orbital parameters are listed in Table 1, it was often argued that such a measurement for the A pulsar via the post-Newtonian gravitomagnetic spin-orbit periastron precession (Barker and O’Connell, 1975a; Damour and Schaefer, 1988; Wex, 1995) would be possible after some years of accurate and continuous timing. Lyne et al. (2004)

Table 1. Relevant Keplerian and post-Keplerian parameters of the binary system PSR J0737-3039A/B (Kramer et al., 2006). The orbital period P_b is measured with a precision of 4×10^{-6} s. The projected semimajor axis is defined as $x \doteq (a_{bc}/c) \sin i$, where a_{bc} is the barycentric semimajor axis (the relative semimajor axis $a = (x_A + x_B)c/\sin i$), c is the speed of light and i is the angle between the plane of the sky, perpendicular to the line-of-sight, and the orbital plane. The eccentricity is e . The best determined post-Keplerian parameter is, to date, the periastron rate $\dot{\omega}$ of the component A. The phenomenologically determined post-Keplerian parameter s , related to the general relativistic Shapiro time delay, is equal to $\sin i$; we have conservatively quoted the largest error in s reported in (Kramer et al., 2006). The other post-Keplerian parameter related to the Shapiro delay, which is used in the text, is r .

P_b (d)	x_A (s)	x_B (s)	e	$\dot{\omega}$ (deg yr $^{-1}$)	s	r (μ s)
0.10225156248(5)	1.415032(1)	1.5161(16)	0.0877775(9)	16.89947(68)	0.99974(39)	6.21(33)

write: “Deviations from the value predicted by general relativity may be caused by contributions from spin-orbit coupling (Barker and O’Connell, 1975b), which is about an order of magnitude larger than for PSR B1913+16. This potentially will allow us to measure the moment of inertia of a neutron star for the first time (Damour and Schaefer, 1988; Wex, 1995).”

According to Lattimer and Schutz (2005), “measurement of the spin-orbit perihelion advance seems possible.”

In (Kramer et al., 2006) we find: “A future determination of the system geometry and the measurement of two other PK parameters at a level of precision similar to that for $\dot{\omega}$, would allow us to measure the moment of inertia of a neutron star for the first time (Damour and Schaefer, 1988; Wex, 1995). [...] this measurement is potentially very difficult [...] The double pulsar [...] would also give insight into the nature of super-dense matter.”

In (Damour, 2007) it is written: “It was then pointed out (Damour and Schaefer, 1988) that this gives, in principle, and indirect way of measuring the moment of inertia of neu-

tron stars [...]. However, this can be done only if one measures, besides¹ k , two other PK parameters with 10^{-5} accuracy. A rather tall order which will be a challenge to meet.”

Some more details are released by Kramer et al. (2005): “[...] a potential measurement of this effect allows the moment of inertia of a neutron star to be determined for the first time (Damour and Schaefer, 1988). If two parameters, e.g. the Shapiro parameter s and the mass ratio \mathcal{R} , can be measured sufficiently accurate, an expected $\dot{\omega}_{\text{exp}}$ can be computed from the intersection point.”

Here we will examine with some more details the conditions which would make feasible to measure I_A at 10% or better in the PSR J0737-3039A/B system in view of the latest timing results (Kramer et al., 2006). In particular, we will show how important the impact of the mismodelling in the known precessional effects affecting the periastron rate of PSR J0737-3039A/B is if other effects must be extracted from such a post-Keplerian parameter. Such an analysis will turn out to be useful also for purposes other than measuring gravitomagnetism like, e.g., putting more severe and realistic constraints on the parameters entering various models of modified gravity (See Section 3.). Indeed, in doing that for, e.g., a uniform cosmological constant Λ Jetzer and Sereno (2006) took into account only the least-square covariance sigma of the estimated periastron rate ($6.8 \times 10^{-4} \text{ deg yr}^{-1}$): the systematic bias due to the first post-Newtonian (1PN) periastron precession was neglected. Concerning the use of the mean anomaly \mathcal{M} for testing 1PN effects on it in pulsar systems, see Section 4..

2.1. The Systematic Uncertainty in the 1PN Periastron Precession

By assuming $I \approx 10^{38} \text{ kg m}^2$ (Morrison et al., 2004; Bejger et al., 2005), the gravitomagnetic spin-orbit periastron precession is about $\dot{\omega}_{\text{GM}} \approx 10^{-4} \text{ deg yr}^{-1}$, while the error $\delta\dot{\omega}_{\text{meas}}$ with which the periastron rate is phenomenologically estimated from timing data is currently $6.8 \times 10^{-4} \text{ deg yr}^{-1}$ (Kramer et al., 2006). In order to measure the gravitomagnetic effect—and, in principle, any other dynamical feature affecting the periastron— $\delta\dot{\omega}_{\text{meas}}$ is certainly of primary importance, but it is not the only source of error to be carefully considered: indeed, there are other terms contributing to the periastron precession (first and second post-Newtonian, quadrupole, spin-spin (Barker and O’Connell, 1975a; Damour and Schaefer, 1988; Wex, 1995)) which must be subtracted from $\dot{\omega}_{\text{meas}}$, thus introducing a further systematic uncertainty due to the propagation of the errors in the system’s parameters entering their analytical expressions. A preliminary analysis of such aspects, can be found in (Lattimer and Schutz, 2005). However, apart from the fact that its authors make use of the value for i measured with the scintillation method (Coles et al., 2005) which is highly uncertain for the reasons given below, in using the third Kepler law to determine the sum of the masses they also confound the relative projected semimajor axis² $a \sin i$ (see eq. (3)) with the barycentric projected semimajor axis x , which is the true measurable quantity from timing data, so that their analysis cannot be considered reliable. The semimajor axis a of the relative motion of A with respect to B in a binary system is just the sum of the semimajor axes a_{bc} of A and B with respect to the system’s barycenter, i.e. $a = a_{\text{bc}}^A + a_{\text{bc}}^B$.

¹The parameter k is directly related to the periastron precession $\dot{\omega}$.

²Our a must not be confused with the one used in (Lattimer and Schutz, 2005) which is, in fact, a_{bc} .

Let us, now, consider the largest contribution to the periastron rate, i.e. the 1PN precession (Damour and Deruelle, 1986; Damour and Taylor, 1992)

$$\dot{\omega}_{1\text{PN}} = \frac{3}{(1-e^2)} \left(\frac{P_b}{2\pi} \right)^{-5/3} (T_\odot M)^{2/3}, \quad (1)$$

where $T_\odot = G\mathfrak{M}_\odot/c^3$ and $M = m_A + m_B$, in units of Solar mass \mathfrak{M}_\odot . It is often referred to as gravitoelectric in the weak-field and slow-motion approximation: in the context of the Solar System it is the well known Einstein Mercury's perihelion precession of about 43 arcsec cy^{-1} . Thus,

$$\begin{cases} \dot{\omega}_{\text{GM}} = \dot{\omega}_{\text{meas}} - \dot{\omega}_{1\text{PN}} - \dot{\omega}_{2\text{PN}}, \\ \delta\dot{\omega}_{\text{GM}} \leq \delta\dot{\omega}_{\text{meas}} + \delta\dot{\omega}_{1\text{PN}} + \delta\dot{\omega}_{2\text{PN}} \end{cases} \quad (2)$$

The sum of the masses M enters eq. (1); as we will see, this implies that the relative semimajor axis a is required as well. For consistency reasons, the values of such parameters used to calculate eq. (1) should have been obtained independently of the periastron rate itself. We will show that, in the case of PSR J0737-3039A/B, it is possible.

Let us start from the relative semimajor axis

$$a = (1 + \mathcal{R}) \left(\frac{cx_A}{\sin i} \right) = 8.78949386 \times 10^8 \text{ m}. \quad (3)$$

It is built in terms of \mathcal{R} , the projected semimajor axis x_A and $\sin i$; the phenomenologically estimated post-Keplerian parameter s determining the shape of the logarithmic Shapiro time delay can be identified with $\sin i$ in GTR and the ratio $\mathcal{R} \doteq x_B/x_A$ has been obtained from the phenomenologically determined projected semimajor axes, being equal to the ratio of the masses in any Lorentz-invariant theory of gravity (Damour and Deruelle, 1985; Damour and Schaefer, 1988; Damour and Taylor, 1992)

$$\mathcal{R} = \frac{m_A}{m_B} + \mathcal{O} \left(\frac{v^4}{c^4} \right). \quad (4)$$

The uncertainty in a can be conservatively evaluated as

$$\delta a \leq \delta a|_{\mathcal{R}} + \delta a|_s + \delta a|_{x_A}, \quad (5)$$

with

$$\begin{cases} \delta a|_{\mathcal{R}} \leq \left(\frac{cx_A}{s} \right) \delta \mathcal{R} = 466,758 \text{ m}, \\ \delta a|_s \leq a \left(\frac{\delta s}{s} \right) = 342,879 \text{ m}, \\ \delta a|_{x_A} \leq a \left(\frac{\delta x_A}{x_A} \right) = 621 \text{ m}. \end{cases} \quad (6)$$

Thus,

$$\delta a \leq 810,259 \text{ m}. \quad (7)$$

eq. (7) yields a relative uncertainty of

$$\frac{\delta a}{a} = 9 \times 10^{-4}. \quad (8)$$

It is important to note that x_B , via \mathcal{R} , and s have a major impact on the overall uncertainty in a ; our estimate has to be considered as conservative because we adopted for δs the largest value quoted in (Kramer et al., 2006). In regard to the inclination, we did not use the more precise value for i obtained from scintillation measurements in³ (Coles et al., 2005) because it is inconsistent with that derived from timing measurements (Kramer et al., 2006). Moreover, the scintillation method is model-dependent and it is not only based on a number of assumptions about the interstellar medium, but it is also much more easily affected by various other effects. However, we will see that also x_A has a non-negligible impact. Finally, let us note that we purposely linearly summed up the individual sources of errors in view of the existing correlations among the various estimated parameters (Kramer et al., 2006).

Let us, now, determine the sum of the masses: recall that it must not come from the periastron rate itself. One possibility is to use the phenomenologically determined orbital period P_b and the third Kepler law getting⁴

$$G\mathfrak{M} = a^3 \left(\frac{2\pi}{P_b} \right)^2. \quad (9)$$

With eq. (3) and eq. (9) we can, now, consistently calculate eq. (1) getting

$$\dot{\omega}_{1\text{PN}} = \frac{3}{(1-e^2)} \left(\frac{x_A + x_B}{s} \right)^2 \left(\frac{2\pi}{P_b} \right)^3 = 16.90410 \text{ deg yr}^{-1}; \quad (10)$$

in this way the 1PN periastron precession is written in terms of the four Keplerian parameters P_b , e , x_A , x_B and of the post-Keplerian parameter s . The mismodeling in them yields

$$\left\{ \begin{array}{l} \delta\dot{\omega}_{1\text{PN}}|_{x_B} \leq 2\dot{\omega}_{1\text{PN}} \left[\frac{\delta x_B}{(x_A + x_B)} \right] = 0.01845 \text{ deg yr}^{-1}, \\ \delta\dot{\omega}_{1\text{PN}}|_s \leq 2\dot{\omega}_{1\text{PN}} \left(\frac{\delta s}{s} \right) = 0.01318 \text{ deg yr}^{-1}, \\ \delta\dot{\omega}_{1\text{PN}}|_{x_A} \leq 2\dot{\omega}_{1\text{PN}} \left[\frac{\delta x_A}{(x_A + x_B)} \right] = 1 \times 10^{-5} \text{ deg yr}^{-1}, \\ \delta\dot{\omega}_{1\text{PN}}|_e \leq 2e\dot{\omega}_{1\text{PN}} \left[\frac{\delta e}{(1-e^2)} \right] = 2 \times 10^{-6} \text{ deg yr}^{-1}, \\ \delta\dot{\omega}_{1\text{PN}}|_{P_b} \leq 3\dot{\omega}_{1\text{PN}} \left(\frac{\delta P_b}{P_b} \right) = \mathcal{O}(10^{-8}) \text{ deg yr}^{-1}. \end{array} \right. \quad (11)$$

Thus, the total uncertainty is

$$\delta\dot{\omega}_{1\text{PN}} \leq 0.03165 \text{ deg yr}^{-1}, \quad (12)$$

which maps into a relative uncertainty of

$$\frac{\delta\dot{\omega}_{1\text{PN}}}{\dot{\omega}_{1\text{PN}}} = 1.8 \times 10^{-3}. \quad (13)$$

³See also (Lyutikov and Thompson, 2005).

⁴In principle, also the 1PN correction to the third Kepler law calculated by (Damour and Deruelle, 1986) should be included, but it does not change the error estimate presented here.

As a consequence, we have the important result

$$\Delta\dot{\omega} \doteq \dot{\omega}_{\text{meas}} - \dot{\omega}_{\text{1PN}} = (-0.00463 \pm 0.03233) \text{ deg yr}^{-1}. \quad (14)$$

Every attempt to measure or constrain effects predicted by known Newtonian and post-Newtonian physics (like, e.g., the action of the quadrupole mass moment or the gravitomagnetic field), or by modified models of gravity, for the periastron of the PSR J0737-3039A/B system must face with the bound of eq. (14).

Should we decide to use both the post-Keplerian parameters related to the Shapiro delay (Damour and Deruelle, 1986; Damour and Taylor, 1992)

$$\begin{cases} r = T_{\odot} m_{\text{B}}, \\ s = x_{\text{A}} \left(\frac{P_{\text{b}}}{2\pi} \right)^{-2/3} T_{\odot}^{-1/3} M^{2/3} m_{\text{B}}^{-1}, \end{cases} \quad (15)$$

for determining the sum of the masses, we would have, with eq. (3),

$$\dot{\omega}_{\text{1PN}} = \frac{3}{(1-e^2)} \left(\frac{P_{\text{b}}}{2\pi} \right)^{3/2} \left(\frac{r}{x_{\text{A}}} \right)^{9/4} \frac{s^{19/4}}{(x_{\text{A}} + x_{\text{B}})^{5/2}}, \quad (16)$$

which yields

$$\dot{\omega}_{\text{1PN}} = 17.25122 \pm 2.11819 \text{ deg yr}^{-1}. \quad (17)$$

The major source of uncertainty is r , with $2.06264 \text{ deg yr}^{-1}$; the bias due to the other parameters is about the same as in the previous case.

Let us, now, consider the second post-Newtonian contribution to the periastron precession (Damour and Schaefer, 1988; Wex, 1995)

$$\dot{\omega}_{\text{2PN}} = \frac{3(G\mathfrak{M})^{5/2}}{c^4 a^{7/2} (1-e^2)^2} \left\{ \left[\frac{13}{2} \left(\frac{m_{\text{A}}^2 + m_{\text{B}}^2}{M^2} \right) + \frac{32}{3} \left(\frac{m_{\text{A}} m_{\text{B}}}{M^2} \right) \right] \right\}, \quad (18)$$

up to terms of order $\mathcal{O}(e^2)$. For our system it amounts to $4 \times 10^{-4} \text{ deg yr}^{-1}$, so that it should be taken into account in $\Delta\dot{\omega}$. However, it can be shown that the bias induced by the errors in \mathfrak{M} and a amounts to $4 \times 10^{-6} \text{ deg yr}^{-1}$, thus affecting the gravitomagnetic precession at the percent level.

2.2. Summary and Discussion

O'Connell (2004), aware of the presence of other non-gravitomagnetic contributions to the pulsar's periastron rate, proposed to try to measure the gravitomagnetic spin-orbit precession of the orbital angular momentum (Barker and O'Connell, 1975a) (analogous to the Lense-Thirring node precession in the limit of a test particle orbiting a massive body) which is not affected by larger gravitoelectric contributions. However, its magnitude is $\approx (10^{-4} \text{ deg yr}^{-1}) \sin \psi$, where ψ is the angle between the orbital angular momentum and the pulsar's spin; thus, it would be negligible in the PSR J0737-3039A/B system because of the near alignment between such vectors (Stairs et al., 2006), in agreement with the observed lack of profile variations (Manchester et al., 2005; Kramer et al., 2006).

In regard to the measurement of the moment of inertia of the component A via the gravitomagnetic periastron precession, our analysis has pointed out that the bias due to the mismodelling in the 1PN gravitoelectric contribution to periastron precession—expressed in terms of the phenomenologically measured parameters P_b, e, x_A, x_B, s —is the most important systematic error exceeding the expected gravitomagnetic rate, at present, by two orders of magnitude; the major sources of uncertainty in it are x_B and s , which should be measured three orders of magnitude better than now to reach the 10% goal. The projected semimajor axis x_A of A, if known one order of magnitude better than now, would induce a percent-level bias. Instead, expressing the 1PN gravitoelectric periastron rate in terms of P_b, e, x_A, x_B, s, r would be definitely not competitive because the improvement required for r would amount to five orders of magnitude at least. We prefer not to speculate now about the size of the improvements in timing of the PSR J0737-3039A/B system which could be achieved in future. Since the timing data of B are required as well for x_B and in view of the fact that B appears as a strong radio source only for two intervals, each of about 10-min duration, while its pulsed emission is rather weak or even undetectable for most of the remainder of the orbit (Lyne et al., 2004; Burgay et al., 2005), the possibility of reaching in a near future the required accuracy to effectively constrain I_A to 10% level or better should be considered with more skepticism than done so far. Another analysis on this topic has been recently performed by Kramer and Wex (2009).

3. Testing a Uniform Cosmological Constant and the DGP Gravity with the Pulsar J0737-3039A

Since, at present, the only reason why the cosmological constant⁵ Λ is believed to be nonzero relies upon the observed acceleration of the universe (Riess et al., 1998; Perlmutter et al., 1999), i.e. just the phenomenon for which Λ was introduced (again), it is important to find independent observational tests of the existence of such an exotic component of the space-time.

Here we put on the test the hypothesis that $\Lambda \neq 0$, where Λ is the uniform cosmological constant of the Schwarzschild-de Sitter (Stuchlik, 1999) (or Kottler (Kottler, 1918)) spacetime, by suitably using the latest determinations of the parameters (see Table 1) of the double pulsar PSR J0737-3039A/B system.

The approach followed here consists in deriving analytical expressions \mathcal{O}_Λ for the effects induced by Λ on some quantities for which empirical values $\mathcal{O}_{\text{meas}}$ determined from fitting the timing data exist. By taking into account the known classical and relativistic effects $\mathcal{O}_{\text{known}}$ affecting such quantities, the discrepancy $\Delta\mathcal{O} = \mathcal{O}_{\text{meas}} - \mathcal{O}_{\text{known}}$ is constructed and attributed to the action of Λ , which was not modelled in the pulsar data processing. Having some $\Delta\mathcal{O}$ and \mathcal{O}_Λ at hand, a suitable combination \mathcal{C} , valid just for the case $\Lambda \neq 0$, is constructed out of them in order to compare $\mathcal{C}_{\text{meas}}$ to \mathcal{C}_Λ : if the hypothesis $\Lambda \neq 0$ is correct, they must be equal within the errors. Here we will use the anomalistic period P_b and the periastron precession $t\omega$ for which purely phenomenological determinations exist in such a way that our \mathcal{C} is the ratio of $\Delta\dot{\omega}$ to ΔP_b ; as we will see, this observable is

⁵See (Calder and Lahav, 2008) and references therein for an interesting historical overview.

independent of Λ but, at the same time, it retains a functional dependence on the system's parameters peculiar to the Λ -induced force and of any other Hooke-like forces.

This Section complements (Iorio, 2008a) in which a similar test was conducted in the Solar System by means of the latest determinations of the secular precessions of the longitudes of the perihelia of several planets. The result by Iorio (2008a) was negative for the Schwarzschild-de Sitter spacetime with uniform Λ ; as we will see, the same conclusion will be traced here in Section 3.1.1..

A complementary approach to explain the cosmic acceleration without resorting to dark energy was followed by Dvali, Gabadadze and Porrati (DGP) in their braneworld modified model of gravity (Dvali et al., 2000). Among other things, it predicts effects which could be tested on a local, astronomical scale. In (Iorio, 2008a) a negative test in the Solar System was reported; as we will see in Section 3.2., PSR J0737-3039A/B confirms such a negative outcome at a much more stringent level.

The overview and the conclusions are in Section 3.3..

3.1. The Impact of Λ on the Periastron and the Orbital Period

The Schwarzschild-de Sitter metric induces an extra-acceleration⁶ (Rindler, 2001)

$$\mathbf{A}_\Lambda = \frac{1}{3}\Lambda c^2 \mathbf{r}, \quad (19)$$

where c is the speed of light; eq. (47), in view of the extreme smallness of the assumed nonzero value cosmological constant ($\Lambda \approx 10^{-52} \text{ m}^{-2}$), can be treated perturbatively with the standard techniques of celestial mechanics. In (Kerr et al., 2003) the secular precession of the pericentre of a test particle around a central body of mass \mathfrak{M} was found to be

$$\dot{\omega}_\Lambda = \frac{\Lambda c^2}{2n} \sqrt{1 - e^2}, \quad (20)$$

where

$$n \doteq \sqrt{\frac{G\mathfrak{M}}{a^3}} \quad (21)$$

is the Keplerian mean motion; a and e are the semimajor axis and the eccentricity, respectively, of the test particle's orbit. Concerning a binary system, in (Jetzer and Sereno, 2006) it was shown that the equations for the relative motion are those of a test particle in a Schwarzschild-de Sitter space-time with a source mass equal to the total mass of the two-body system, i.e. $\mathfrak{M} = m_A + m_B$. Thus, eq. (20) is valid in our case; a is the semi-major axis of the relative orbit.

Following the approach by Jetzer and Sereno (2006), we will now compute P_Λ , i.e. the contribution of Λ to the orbital period. One of the six Keplerian orbital elements in terms of which it is possible to parameterize the orbital motion in a binary system is the mean anomaly \mathcal{M} defined as $\mathcal{M} \doteq n(t - T_0)$, where n is the mean motion and T_0 is the time of pericenter passage. The mean motion $n \doteq 2\pi/P_b$ is inversely proportional to the time elapsed between two consecutive crossings of the pericenter, i.e. the anomalistic period

⁶The present test is valid for all exotic Hooke-type forces of the form Cr (Calder and Lahav, 2008), with C arbitrary nonzero constant.

P_b . In Newtonian mechanics, for two point-like bodies, n reduces to the usual Keplerian expression $n = 2\pi/P_b$. In many binary systems, as in the double pulsar one, the period P_b is accurately determined in a phenomenological, model-independent way, so that, in principle, it accounts for all the dynamical features of the system, not only those coming from the Newtonian point-like terms, within the measurement precision.

The Gauss equation for the variation of the mean anomaly, in the case of an entirely radial disturbing acceleration \mathcal{A}_r like eq. (47), is (Roy, 1988)

$$\frac{d\mathcal{M}}{dt} = n - \frac{2}{na}\mathcal{A}_r \left(\frac{r}{a}\right) + \frac{(1-e^2)}{nae}\mathcal{A}_r \cos f, \quad (22)$$

where f is the true anomaly, reckoned from the pericenter. Using the eccentric anomaly E , defined as

$$\mathcal{M} = E - e \sin E, \quad (23)$$

turns out to be more convenient. The unperturbed Keplerian ellipse, on which the right-hand-side of eq. (22) must be evaluated, is

$$r = a(1 - e \cos E); \quad (24)$$

by using eq. (47) and

$$\begin{cases} \frac{d\mathcal{M}}{dE} = 1 - e \cos E, \\ \cos f = \frac{\cos E - e}{1 - e \cos E}, \end{cases} \quad (25)$$

eq. (22) becomes

$$\begin{aligned} \frac{dE}{dt} &= \frac{n}{(1 - e \cos E)} \left\{ 1 - \frac{\Lambda c^2}{3n^2} \left[2(1 - e \cos E)^2 - \right. \right. \\ &\quad \left. \left. - \frac{(1 - e^2)}{e} (\cos E - e) \right] \right\}. \end{aligned} \quad (26)$$

Since $\Lambda c^2/3n^2 \approx 10^{-29}$ from eq. (26) it can be obtained

$$\begin{aligned} P_b &\simeq \int_0^{2\pi} \frac{(1 - e \cos E)}{n} \left\{ 1 + \frac{\Lambda c^2}{3n^2} \left[2(1 - e \cos E)^2 - \right. \right. \\ &\quad \left. \left. - \frac{(1 - e^2)}{e} (\cos E - e) \right] \right\} dE, \end{aligned} \quad (27)$$

which integrated yields that

$$P_b = P_b + P_\Lambda \quad (28)$$

with

$$P_\Lambda = \frac{\pi \Lambda c^2 (7 + 3e^2)}{3n^3}. \quad (29)$$

3.1.1. Combining the Periastron and the Orbital Period

For the sake of convenience, from Section 2.1. let us recall the general relativistic expressions of the post-Keplerian parameters r , s and $\dot{\omega}$;

$$\begin{cases} r = T_{\odot} m_B, \\ s = x_A \left(\frac{P_b}{2\pi} \right)^{-2/3} T_{\odot}^{-1/3} M^{2/3} m_B^{-1}, \\ \dot{\omega}_{\text{1PN}} = \frac{3}{1-e^2} \left(\frac{P_b}{2\pi} \right)^{-5/3} (T_{\odot} M)^{2/3}. \end{cases} \quad (30)$$

By means of

$$a = \frac{c}{s}(x_A + x_B) \quad (31)$$

and of the equations for r and s it is possible to express P_b and $\dot{\omega}_{\text{1PN}}$ in terms of P_b and of the phenomenologically determined Keplerian and post-Keplerian parameters x_A, x_B, r, s as

$$\begin{cases} P_b = 2 \left(\frac{2}{P_b} \right)^{1/2} [\pi(x_A + x_B)]^{3/2} \left(\frac{x_A}{r} \right)^{3/4} s^{-9/4}, \\ \dot{\omega}_{\text{1PN}} = \frac{3sr}{x_A(1-e^2)} \left(\frac{P_b}{2\pi} \right)^{-1}. \end{cases} \quad (32)$$

In such a way we can genuinely compare them to P_b and $\dot{\omega}$ because they do not contain quantities obtained from the third Kepler law and the general relativistic periastron precession themselves; moreover, we have expressed the sum of the masses entering both P_b and $\dot{\omega}_{\text{1PN}}$ in terms of r and s , thus avoiding any possible reciprocal imprinting between the third Kepler law and the periastron rate. At this point it is possible to construct

$$R \doteq \frac{\Delta\dot{\omega}}{\Delta P}, \quad (33)$$

with

$$\begin{cases} \Delta\dot{\omega} = \dot{\omega} - \dot{\omega}_{\text{1PN}}, \\ \Delta P = P_b - P_b; \end{cases} \quad (34)$$

note that

$$R = R(P_b, x_A, x_B, e; \dot{\omega}, r, s). \quad (35)$$

By attributing $\Delta\dot{\omega}$ and ΔP to the action of Λ , not modelled into the routines used to fit the PSR J0737-3039A/B timing data, it is possible to compare R to

$$R_{\Lambda} \doteq \frac{\dot{\omega}_{\Lambda}}{P_{\Lambda}} = \frac{3\sqrt{1-e^2} P_b r^{3/2} s^{9/2}}{4\pi^2(7+3e^2)(x_A + x_B)^3 x_A^{3/2}} \quad (36)$$

and see if eq. (33) and eq. (36) are equal within the errors. Note that eq. (36) is independent of Λ and, by definition, is able to test the hypothesis that $\Lambda \neq 0$. From Table 1 it turns out

$$R_{\Lambda} = (3.4 \pm 0.3) \times 10^{-8} \text{ s}^{-2}; \quad (37)$$

R_Λ is a well determined quantity, different from zero at about 11 sigma level. In regard to R we have

$$\begin{cases} \Delta\dot{\omega} = -0.3 \pm 2.1 \text{ deg yr}^{-1}, \\ \Delta P = 59 \pm 364 \text{ s}, \end{cases} \quad (38)$$

so that

$$|R| = (0.3 \pm 4) \times 10^{-11} \text{ s}^{-2}; \quad (39)$$

R is compatible with zero in such a way that its range does not overlap with the one of R_Λ : indeed, the upper bound on R is three orders of magnitude smaller than the lower bound on R_Λ . Thus, we must conclude that⁷

$$R \neq R_\Lambda. \quad (40)$$

Concerning the released uncertainties in R and R_Λ , they must be considered as upper bounds since they have been conservatively computed by linearly adding the individual biased terms due to $\delta P_b, \delta\dot{\omega}, \delta e, \delta x_A, \delta x_B, \delta r, \delta s$ in order to account for the existing correlations (Kramer et al., 2006) among them.

The results of the present study confirm those obtained in the Solar System by taking the ratio of the estimated corrections to the standard Newtonian/Einsteinian precessions of the longitude of the perihelia ϖ for different pairs of planets (Iorio, 2008a). It would be very interesting to devise analogous tests involving other observables (lensing, time delay) affected by Λ as well recently computed in, e.g., (Ruggiero, 2009; Sereno, 2008).

3.2. The Dvali-Gabadadze-Porrati Braneworld Model

The approach previously outlined for Λ can be followed also for the DGP braneworld model (Dvali et al., 2000) which recently received great attention from an observational point of view (Dvali et al., 2003; Iorio, 2008b).

The preliminary confrontations with data so far performed refer to the perihelia of the Solar System planets. Indeed, DGP gravity predicts an extra-precession of the pericentre of a test particle (Lue and Starkman, 2003; Iorio, 2005a)

$$\dot{\omega}_{\text{DGP}} = \mp \frac{3}{8} \left(\frac{c}{r_0} \right) \left(1 - \frac{13}{32} e^2 \right), \quad (41)$$

where the signs \mp are related to the two different cosmological branches of the model and r_0 is a free-parameter set to about 5 Gpc by Type IA Supernovae data, independent of the orbit's semimajor axis. The predicted precessions of about $10^{-4} \text{ arcsec cy}^{-1}$ were found to be compatible with the estimated corrections to the usual apsidal precessions of planets considered one at a time separately (Iorio, 2008b), but marginally incompatible with the ratio of them for some pairs of inner planets (Iorio, 2007b).

The effects of DGP model on the orbital period is (Iorio, 2006)

$$P_{\text{DGP}} = \mp \frac{11}{8} \pi \left(\frac{c}{r_0} \right) \frac{a^3 (1 - e^2)^2}{G\mathfrak{M}}. \quad (42)$$

⁷In principle, also the 1PN correction to the third Kepler law (Damour and Deruelle, 1986) should be included in ΔP , but it does not change the result.

From eq. (41) and eq. (42) it is possible to construct

$$R_{\text{DGP}} \doteq \frac{\dot{\omega}_{\text{DGP}}}{P_{\text{DGP}}}, \quad (43)$$

which, expressed in terms of the phenomenologically determined parameters of PSR J0737-3039A/B, becomes

$$R_{\text{DGP}} = \frac{3 \left(1 - \frac{13}{32}e^2\right) P_{\text{b}} r^{3/2} s^{9/2}}{22\pi(1 - e^2) (x_{\text{A}} + x_{\text{B}})^3 x_{\text{A}}^{3/2}}. \quad (44)$$

Putting the figures of Table eq. (1) into eq. (44) and computing the uncertainty as done in the case of Λ yields

$$R_{\text{DGP}} = (1.4 \pm 0.1) \times 10^{-7} \text{ s}^{-2}. \quad (45)$$

As can be noted, the lower bound of R_{DGP} is four orders of magnitude larger than the upper bound of R , so that we must conclude that, also in this case,

$$R \neq R_{\text{DGP}}. \quad (46)$$

The outcome by Iorio (2007b) is, thus, confirmed at a much more stringent level.

An analysis of type Ia supernovae (SNe Ia) data disfavoring DGP model can be found in (Bento et al., 2005).

3.3. Summary and Discussion

In this Section we used the most recent determinations of the orbital parameters of the double pulsar binary system PSR J0737-3039A/B to perform local tests of two complementary approaches to the issue of the observed acceleration of the universe: the uniform cosmological constant Λ in the framework of the known general relativistic laws of gravity and the multidimensional braneworld model by Dvali, Gabadadze and Porrati which, instead, resorts to a modification of the currently known laws of gravity. Since, at present, there are no observational evidences for such theoretical schemes other than just the cosmological phenomenon for which they were introduced, it is important to put them on the test independently of the cosmological acceleration itself. It is worthwhile noting that the results for Λ hold also for any other Hooke-like additional force proportional to r .

To this aim, we considered the phenomenologically determined the periastron precession $\dot{\omega}$ and the orbital period P_{b} of PSR J0737-3039A/B by contrasting them to the predicted 1PN periastron rate $\dot{\omega}_{\text{1PN}}$ and the Keplerian period P_{b} . With such discrepancies we constructed the ratio $R \doteq \Delta\omega/\Delta P$ by finding it compatible with zero: $|R| = (0.3 \pm 4) \times 10^{-11} \text{ s}^{-2}$. Then, we compared R to the predicted ratios for the effects of Λ and the DGP gravity-not modeled in the pulsar data processing-on the periastron rate and the orbital period by finding $R_{\Lambda} = (3.4 \pm 0.3) \times 10^{-8} \text{ s}^{-2}$ and $R_{\text{DGP}} = (1.4 \pm 0.1) \times 10^{-7} \text{ s}^{-2}$, respectively. Thus, the outcome of such a local test is neatly negative, in agreement with other local tests recently performed in the Solar System by taking the ratio of the non-Newtonian/Einsteinian rates of the perihelia for several pairs of planets.

4. The 1PN Secular Effects on the Mean Anomaly in Binary Pulsar Systems

According to the Einstein's General Theory of Relativity (GTR), the post-Newtonian gravitoelectric two-body acceleration of order $\mathcal{O}(c^{-2})$ (1PN) is, in the post-Newtonian centre of mass frame (see (Damour and Deruelle, 1985) and, e.g., (Portilla and Villareal, 2004) and references therein)

$$\mathbf{a}^{\text{GE}} = \frac{G\mathfrak{M}}{c^2 r^3} \left\{ \left[\frac{G\mathfrak{M}}{r} (4 + 2\nu) - (1 + 3\nu)v^2 + \frac{3\nu}{2r^2} (\mathbf{r} \cdot \mathbf{v})^2 \right] \mathbf{r} + (\mathbf{r} \cdot \mathbf{v})(4 - 2\nu)\mathbf{v} \right\}, \quad (47)$$

where \mathbf{r} and \mathbf{v} are the relative position and velocity vectors, respectively, G is the Newtonian constant of gravitation, c is the speed of light, m_1 and m_2 are the rest masses of the two bodies, $\mathfrak{M} \doteq m_1 + m_2$ and $\nu \doteq m_1 m_2 / m^2 < 1$.

Let us recall from Section 3.1. that the orbital phase can be characterized by the mean anomaly \mathcal{M} defined as

$$\mathcal{M} \doteq n(t - T_0), \quad (48)$$

where the unperturbed mean motion n is defined as

$$n \doteq \frac{2\pi}{P_b}. \quad (49)$$

In it P_b is the anomalistic period, i.e. the time elapsed between two consecutive pericentre crossings, which is $2\pi\sqrt{a^3/G\mathfrak{M}}$ for an unperturbed Keplerian ellipse, and T_0 is the date of a chosen pericentre passage.

The variation of the mean anomaly can be written, in general, as⁸

$$\frac{d\mathcal{M}}{dt} = n - 2\pi \left(\frac{\dot{P}_b}{P_b^2} \right) (t - T_0) - \frac{2\pi}{P_b} \left(\frac{dT_0}{dt} \right). \quad (50)$$

The second term of the right-hand side of eq. (50) accounts for any possible variation of the anomalistic period. The third term, induced by any small perturbing acceleration with respect to the Newtonian monopole, whether relativistic or not, is the change of the time of the pericentre passage, which we will define as

$$\frac{d\xi}{dt} \doteq -\frac{2\pi}{P_b} \left(\frac{dT_0}{dt} \right). \quad (51)$$

It can be calculated with the aid of the Gauss⁹ perturbative equation (Roy, 1988)

$$\frac{d\xi}{dt} = -\frac{2}{na} \mathcal{A}_r \left(\frac{r}{a} \right) - \sqrt{1 - e^2} \left(\frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt} \right), \quad (52)$$

where i, Ω are the inclination and the longitude of the ascending node, respectively, of the orbit. In order to obtain the secular effects, we must evaluate the right-hand-side of eq.

⁸See also Lin-Sen (2010).

⁹For a different approach based on a modified form of the Lagrange planetary equations see (Calura et al., 1997).

(52) on the unperturbed Keplerian ellipse and, then, average the result over one orbital revolution.

We will now consider eq. (47) as perturbing acceleration. Let us start with the first term of the right-hand-side of eq. (52). By defining

$$\begin{cases} A & \doteq \frac{(G\mathfrak{M})^2}{c^2}(4 + 2\nu), \\ B & \doteq -\frac{G\mathfrak{M}}{c^2}(1 + 3\nu), \\ C & \doteq \frac{G\mathfrak{M}}{c^2}\left(4 - \frac{\nu}{2}\right), \end{cases} \quad (53)$$

it is possible to obtain from eq. (47)

$$\mathcal{A}_r^{\text{GE}} = \frac{A}{r^3} + B\left(\frac{v^2}{r^2}\right) + C\left(\frac{\dot{r}}{r^2}\right). \quad (54)$$

Now the term $-2\mathcal{A}_r r/na^2$, with \mathcal{A}_r given by eq. (54), must be evaluated on the unperturbed Keplerian ellipse characterized by

$$\begin{cases} r & = \frac{a(1-e^2)}{1+e\cos f}, \\ \dot{r} & = \frac{nae\sin f}{\sqrt{1-e^2}}, \\ v^2 & = \frac{n^2a^2}{(1-e^2)}(1 + e^2 + 2e\cos f) \end{cases} \quad (55)$$

where f is the true anomaly, and averaged over one orbital period by means of

$$\frac{dt}{P_b} = \frac{r^2 df}{2\pi a^2 \sqrt{1-e^2}}. \quad (56)$$

Thus,

$$-\left(\frac{2}{na}\mathcal{A}_r^{\text{GE}}\frac{r}{a}\right)\left(\frac{dt}{P_b}\right) = -\frac{1}{na^4\pi\sqrt{1-e^2}}[A + Brv^2 + Cr(\dot{r})^2]df. \quad (57)$$

In the expansion of r in eq. (57) the terms of order $\mathcal{O}(e^4)$ are retained. The final result is

$$\left\langle -\frac{2}{na}\mathcal{A}_r^{\text{GE}}\frac{r}{a} \right\rangle_{P_b} = \frac{nG\mathfrak{M}}{c^2a\sqrt{1-e^2}}H(e; \nu), \quad (58)$$

with

$$\begin{aligned} H \simeq & -2(4 + 2\nu) + (1 + 3\nu)\left(2 + e^2 + \frac{e^4}{4} + \frac{e^6}{8}\right) - \\ & -\left(4 - \frac{\nu}{2}\right)\left(e^2 + \frac{e^4}{4} + \frac{e^6}{8}\right). \end{aligned} \quad (59)$$

The post-Newtonian gravitoelectric secular rate of pericentre is independent of ν and is given by the well known formula

$$\left. \frac{d\omega}{dt} \right|_{\text{GE}} = \frac{3nG\mathfrak{M}}{c^2 a(1-e^2)}, \quad (60)$$

while there are no secular effects on the node.

The final expression for the post-Newtonian secular rate of the mean anomaly is obtained by combining eq. (58)-eq. (60) and by considering that, for a two-body system, it is customarily to write

$$\frac{nG\mathfrak{M}}{c^2} = \left(\frac{P_b}{2\pi} \right)^{-5/3} (T_\odot M)^{2/3}. \quad (61)$$

It is

$$\left. \frac{d\xi}{dt} \right|_{\text{GE}} = -9 \left(\frac{P_b}{2\pi} \right)^{-5/3} (T_\odot M)^{2/3} (1-e^2)^{-1/2} F(e; \nu) \quad (62)$$

with

$$F = \left[\left(1 + \frac{e^2}{3} + \frac{e^4}{12} + \frac{e^6}{24} + \dots \right) - \frac{2}{9}\nu \left(1 + \frac{7}{4}e^2 + \frac{7}{16}e^4 + \frac{7}{32}e^6 + \dots \right) \right]. \quad (63)$$

Note that eq. (62) is negative because eq. (63) is always positive; thus the crossing of the apsidal line occurs at a later time with respect to the Kepler-Newton case.

Note that, for $\nu \rightarrow 0$, i.e. $m_1 \ll m_2$, eq. (62) does not vanish and, for small eccentricities, it becomes

$$\left. \frac{d\xi}{dt} \right|_{\text{GE}} \simeq -\frac{9nGm_2}{c^2 a \sqrt{1-e^2}} \left(1 + \frac{e^2}{3} \right), \quad (64)$$

which could be used for planetary motion in the Solar System (Iorio, 2005b). E.g., for Mercury it yields a secular effect of almost -130 arcsec cy^{-1} . It is important to note that the validity of the present calculations has also been numerically checked by integrating over 200 years the Jet Propulsion Laboratory (JPL) equations of motion of all the planets of the Solar System with and without the gravitoelectric $1/c^2$ terms in the dynamical force models (Estabrook, 1971) in order to single out just the post-Newtonian gravitoelectric effects. They fully agree with eq. (64) (E.M. Standish, private communication, 2004). Another analytical calculation of the post-Newtonian general relativistic gravitoelectric secular rate of the mean anomaly was performed (Rubincam, 1977) in the framework of the Lagrangian perturbative scheme for a central body of mass \mathfrak{M} -test particle system. Rubincam (1977) starts from the space-time line element of the Schwarzschild metric written in terms of the Schwarzschild radial coordinate r' . Instead, eq. (47) and the equations of motion adopted in the practical planetary data reduction at, e.g. JPL, are written in terms of the standard isotropic radial coordinate r related to the Schwarzschild coordinate by $r' = r(1 + G\mathfrak{M}/2c^2 r)^2$. As a consequence, the obtained exact expression

$$\left. \frac{d\xi}{dt} \right|_{\text{GE}}^{(\text{Rubincam})} = \frac{3nG\mathfrak{M}}{c^2 a \sqrt{1-e^2}}, \quad (65)$$

contrary to the pericentre case, agrees neither with eq. (64) nor with the JPL numerical integrations yielding, e.g., a secular advance of $+42$ arcsec cy^{-1} for Mercury. For a better

understanding of such comparisons, let us note that both the numerical analysis by Standish and Rubincam (1977) are based on the $\dot{P}_b = 0$ case; n gets canceled by construction in the Standish calculation, while in (Rubincam, 1977) the numbers are put just into eq. (65), which is the focus of that work.

4.1. Testing Gravitational Theories with Binary Pulsars

In general, in the pulsar's timing data reduction process¹⁰ five Keplerian orbital parameters and a certain number of post-Keplerian parameters are determined with great accuracy in a phenomenological way, independently of any gravitational theory (Wex, 2001; Stairs, 2003). The Keplerian parameters are the projected semimajor axis $x = a \sin i/c$, where i is the angle between the plane of the sky, which is normal to the line of sight and is assumed as reference plane, and the pulsar's orbital plane, the eccentricity e , the orbital period P_b , the time of periastron passage T_0 and the argument of periastron ω_0 at the reference time T_0 . The most commonly used post-Keplerian parameters are the periastron secular advance $\dot{\omega}$, the combined time dilation and gravitational redshift due to the pulsar's orbit γ , the variation of the anomalistic period \dot{P}_b , the range r and the shape s of the Shapiro delay. These post-Keplerian parameters are included in the timing models (Wex, 2001; Stairs, 2003) of the so called Roemer, Einstein and Shapiro $\Delta_R, \Delta_E, \Delta_S$ delays¹¹ occurring in the binary pulsar system¹²

$$\begin{cases} \Delta_R &= x \sin \omega [\cos E - e(1 + \delta_r)] + x \cos \omega \sin E \sqrt{1 - e^2(1 + \delta_\theta)^2}, \\ \Delta_E &= \gamma \sin E, \\ \Delta_S &= -2r \ln\{1 - e \cos E - s[\sin \omega (\cos E - e) + \sqrt{1 - e^2} \cos \omega \sin E]\}, \end{cases} \quad (66)$$

where E is the eccentric anomaly defined as $E - e \sin E = \mathcal{M}$. $\cos E$ and $\sin E$ appearing in eq. (66) can be expressed in terms of \mathcal{M} by means of the following elliptic expansions (Vinti, 1998)

$$\begin{cases} \cos E &= -\frac{e}{2} + \sum_{j=1}^{\infty} \frac{2}{j^2} \frac{d}{de} [J_j(je)] \cos(j\mathcal{M}), \\ \sin E &= \frac{2}{e} \sum_{j=1}^{\infty} \frac{J_j(je)}{j} \sin(j\mathcal{M}), \end{cases} \quad (67)$$

where $J_j(y)$ are the Bessel functions defined as

$$\pi J_j(y) = \int_0^\pi \cos(j\theta - y \sin \theta) d\theta. \quad (68)$$

The relativistic secular advance of the mean anomaly eq. (62) can be accounted for in the pulsar timing modelling by means of eq. (67).

In a given theory of gravity, the post-Keplerian parameters can be written in terms of the mass of the pulsar m_p and of the companion m_c . In general, m_p and m_c are unknown;

¹⁰For all general aspects of the binary pulsar systems see (Wex, 2001; Stairs, 2003) and references therein.

¹¹For the complete expression of the timing models including, e.g., also the delays occurring in the Solar System due to the solar gravity see (Wex, 2001; Stairs, 2003).

¹²The aberration parameters δ_r and δ_θ are not, in general, separately measurable.

this means that the measurement of only one post-Keplerian parameter, say, the periastron advance, cannot be considered as a test of a given theory of gravity because one would not have a theoretically calculated value to be compared with the phenomenologically measured one. In GTR the previously quoted post-Keplerian parameters are (Damour and Deruelle, 1986)

$$\left\{ \begin{array}{lcl} \dot{\omega} & = & 3 \left(\frac{P_b}{2\pi} \right)^{-5/3} (T_\odot M)^{2/3} (1 - e^2)^{-1}, \\ \gamma & = & e \left(\frac{P_b}{2\pi} \right)^{1/3} T_\odot^{2/3} M^{-4/3} m_c (m_p + 2m_c), \\ \dot{P}_b & = & -\frac{192\pi}{5} T_\odot^{5/3} \left(\frac{P_b}{2\pi} \right)^{-5/3} \frac{(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4)}{(1-e^2)^{7/2}} \frac{m_p m_c}{M^{1/3}}, \\ r & = & T_\odot m_c, \\ s & = & x T_\odot^{-1/3} \left(\frac{P_b}{2\pi} \right)^{-2/3} \frac{M^{2/3}}{m_c} \end{array} \right. \quad (69)$$

It is important to note that the relativistic expression of \dot{P}_b in eq. (69), should not be confused with $\dot{\xi}|_{\text{GE}}$ of eq. (62). Indeed, it refers to the shrinking of the orbit due to gravitational wave emission which vanishes in the limit $\nu \rightarrow 0$, contrary to eq. (62) which expresses a different, independent phenomenon. The measurement of two post-Keplerian orbital parameters allows to determine m_p and m_c , assumed the validity of a given theory of gravity¹³. Such values can, then, be inserted in the analytical expressions of the remaining post-Keplerian parameters. If the so obtained values are equal to the measured ones, or the curves for the $2 + N$, with $N \geq 1$, measured post-Keplerian parameters in the $m_p - m_c$ plane all intersect in a well determined (m_p, m_c) point, the theory of gravity adopted is consistent. So, in order to use the pulsar binary systems as valuable tools for testing GTR the measurement of at least three post-Keplerian parameters is required. The number of post-Keplerian parameters which can effectively be determined depends on the characteristics of the particular binary system under consideration. For the pulsar-neutron star PSR B1913+16 system (Hulse and Taylor, 1975) the three post-Keplerian parameters $\dot{\omega}$, γ and \dot{P}_b were measured with great accuracy. For the pulsar-neutron star PSR B1534+12 system (Stairs et al., 2002) the post-Keplerian parameters reliably measured are $\dot{\omega}$, γ , r and s . For the pulsar-pulsar PSR J0737-3039 A+B system the same four post-Keplerian parameters as for PSR B1534+12 are available plus \dot{P}_b and a further constraint on m_p/m_c coming from the measurement of both the projected semimajor axes. On the contrary, in the pulsar-white dwarf binary systems, which are the majority of the binary systems with one pulsar and present almost circular orbits, it is often impossible to measure $\dot{\omega}$ and γ . Up to now, only r and s have been measured, with a certain accuracy, in the PSR B1855+09 system (Kaspi et al., 1994), so that it is impossible to use its data for testing the GTR as previously outlined.

¹³This would still not be a test of the GTR because the masses must be the same for all the theories of gravity, of course.

4.2. The Secular Decrease of the Mean Anomaly and the Binary Pulsars

Let us investigate the magnitude of the mean anomaly precession in some systems including one or two radiopulsars.

For PSR B1913+16 we have (Weisberg and Taylor, 2005) $m_p = 1.4414$, $m_c = 1.3867$, $e = 0.6171338$, $P_b = 0.322997448930$ d. Then, $\nu = 0.2499064$, $F = 1.04459537192$ and $\dot{\xi}|_{\text{GE}} = -10.422159$ deg yr⁻¹. For PSR J0737-3039 A we have $\nu = 0.249721953643$, $F = 0.946329857430$. Thus, $\dot{\xi}|_{\text{GE}} = -47.79$ deg yr⁻¹. This implies that the ratio of the post-Newtonian gravitoelectric secular rate of the mean anomaly to the mean motion amounts to $\sim 10^{-5}$. Let us see if such post-Newtonian shift is detectable from quadratic fits of the orbital phases of the form $\mathcal{M} = a_0 + b_0 t + c_0 t^2$. For PSR B1913+16 the quadratic advance due to the gravitational wave emission over thirty years amounts to ($\dot{P}_b = -2.4184 \times 10^{-12}$)

$$\Delta\mathcal{M} = -\pi \left(\frac{\dot{P}_b|_{\text{gw}}}{P_b^2} \right) (t - T_0)^2 = 0.5 \text{ deg}, \quad (70)$$

with an uncertainty $\delta(\Delta\mathcal{M})$ fixed to 0.0002 deg by $\delta\dot{P}_b = 0.0009 \times 10^{-12}$. The linear shift due to eq. (62) amounts to

$$\Delta\mathcal{M} = \dot{\xi}|_{\text{GE}} (t - T_0) = -312.6647 \text{ deg} \quad (71)$$

over the same time interval. The uncertainty in¹⁴ n amounts to 1×10^{-9} deg yr⁻¹ due to $\delta P_b = 4 \times 10^{-12}$ d. For PSR J0737-3039 A the gravitational wave emission over three years ($\dot{P}_b = -1.20 \times 10^{-12}$) induces a quadratic shift of 0.008 deg, with an uncertainty $\delta(\Delta\mathcal{M})$ fixed to 0.0005 deg by $\delta\dot{P}_b = 0.08 \times 10^{-12}$. The linear shift due to eq. (62) amounts to -143.3700 deg over the same time interval. The uncertainty in n amounts to 7×10^{-7} deg yr⁻¹ due to $\delta P_b = 2 \times 10^{-10}$ d. Thus, it should be possible to extract $\dot{\xi}|_{\text{GE}}$ from the measured coefficient b_0 ; both the corrupting bias due to the uncertainties in the quadratic signature and the errors in n would be negligible.

Measuring $\dot{\xi}|_{\text{GE}}$ as a further post-Keplerian parameter would be very useful in those scenarios in which some of the traditional post-Keplerian parameters are known with a modest precision or, for some reasons, cannot be considered entirely reliable¹⁵. E.g., in the double pulsar system PSR J0737-3039 A+B the parameters r and γ are measured with a relatively low accuracy (Lyne et al., 2004). Moreover, there are also pulsar binary systems in which only the periastron rate has been measured (Kaspi, 1999): in this case the knowledge of another post-Keplerian parameter would allow to determine the masses of the system, although it would not be possible to constraint alternative theories of gravity.

¹⁴The sum of the masses and the semimajor axis entering n are determined from timing data processing independently of \mathcal{M} itself, e.g. from the periastron rate and the projected barycentric semimajor axis.

¹⁵The measured value of the derivative of the orbital period \dot{P}_b is aliased by several external contributions which often limit the precision of the tests of competing theories of gravity based on this post-Keplerian parameter (Stairs, 2003).

4.3. Overview and Discussion

In this Section we have analytically derived for a two-body system in eccentric orbits the secular variation $\dot{\xi}|_{\text{GE}}$ yielding the post-Newtonian general relativistic gravitoelectric part of the precession of the mean anomaly not due to the variation of the orbital period. For a complementary analysis, see Lin-Sen (2010). In the limit of small eccentricities and taking the mass of one of the two bodies negligible, our results have been compared to the outcome of a numerical integration of the post-Newtonian general relativistic gravitoelectric equations of motion of the planets of the Solar System performed by JPL: the agreement between the analytical and numerical calculation is complete. Subsequently, we have investigated the possibility of applying the obtained results to the binary systems in which one pulsar is present. In particular, it has been shown that the variation of the orbital period $\dot{P}_b|_{\text{gw}}$ due to gravitational wave emission and the effect derived by us are different ones. Indeed, the post-Newtonian gravitoelectric precession of the mean anomaly, which is always negative, is related to the secular increase of the time of pericentre passage and occurs even if the orbital period does not change in time. A quadratic fit of the orbital phase of the pulsar would allow to measure $\dot{\xi}|_{\text{GE}}$ because the biases due to the errors in the quadratic shift due to \dot{P}_b and in the linear shift due of the mean motion n are smaller. The use of $\dot{\xi}|_{\text{GE}}$ as a further post-Keplerian parameter would allow to improve and enhance the tests of post-Newtonian gravity especially for those systems in which only few post-Keplerian parameters can be reliably measured.

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